Fractions

When a fraction is written as a decimal, it either terminates or the digits repeat. It terminates when the denominator is a product of 2's and 5's. It is interesting to study how many digits are repeated in the decimal expansion of $1/n$.
For example $1/9 = 0.11111\ldots$ has one $1/11 = 0.09090909\ldots$ has two $1/37 = 0.027027027\ldots$ has three.
If n is a prime, the number of digits that repeat in $1/n$ is always a factor of $n-1$. For $n = 7$, we actually get 6, since

$$1/7 = 0.142857142857142857\ldots$$

Some other primes with this property are 17, 19, 23 and 29:

$$\begin{aligned}
1/17 &= 0.0588235294117647058823552\ldots \\
1/19 &= 0.052631578947368421052 6\ldots \\
1/23 &= 0.04347826086956521739130434\ldots \\
1/29 &= 0.034482758620689655172413793 1034\ldots
\end{aligned}$$

Here are the values of $m/17$ for m from 1 to 5.

$$\begin{aligned}
1/17 &= 0.05882352941176470588\ldots \\
2/17 &= 0.11764705882352941176\ldots \\
3/17 &= 0.17647058823529411764\ldots \\
4/17 &= 0.23529411764705882352\ldots \\
5/17 &= 0.29411764705882352941\ldots
\end{aligned}$$

Some things to think about.

(1) Can you spot any pattern?
(2) Can you predict what 8/17 and 9/17 might be?
(3) What is the next prime n after 29 for which $1/n$ does not repeat before the $(n-1)$th digit?
(4)

$$1/81 = 0.012345679012345679012345679\ldots$$

What happened to the 8?
(5) If $x = 0.(1)^{\cdot} = 0.111111\ldots$, what is the decimal expression for x^2?
(6) If $x = 0.(1)^{\cdot} = 0.111111\ldots$, then $x^3 \approx 0.001371742112482853223593964334705075445816186556927 30\ldots$
How many digits do we have to calculate before the sequence of digits 13717 appear again?
(7) If $x = 0.(01)^{\cdot} = 0.01010101\ldots$, then $x^2 \approx 0.000102030405060708091011121314151617181920\ldots$
Can you predict the full decimal expansion of this number?

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